THE INFLUENCE OF GEOMETRY ON THE BUCKLING OF THIN-WALLED CIRCULAR CYLINDRICAL SHELLS

A THESIS
SUBMITTED TO THE DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS
AND THE COMMITTEE ON THE GRADUATE DIVISION
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
ENGINEER

By
Richard Lloyd Komp
July 1965

Approved for the Department:

ACKNOWLEDGMENT

The research described in this thesis is part of a general study on the buckling of shell bodies being carried out by the Department of Aeronautics and Astronautics at Stanford University. The work was sponsored in part by the U. S. Army Aviation Materiel Laboratories under contract DA 44-177-AMC-258(T), and in part by the U. S. Air Force Office of Scientific Research under contract AF 49(638) 1495.

The author wishes to express his sincere appreciation to Professor Wilfred H. Horton for posing the problem and for his guidance and contributions during the research and preparation of this paper.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

E	Young's modulus of shell material
L	Length of shell
n	Number of buckles
R	Radius of shell
t	Thickness of shell
σ_{cr}	Critial stress
σ_{cl}	Classical critical stress
λ	Buckle wavelength

1. INTRODUCTION

The thin-walled unstiffened circular cylindrical shell is one of the most commonly used elements in general engineering. Knowledge of the behavior of such a body in various environments is therefore of importance to designers. As a consequence, there has been during the last one hundred years, many analytical and experimental studies made on the several pertinent aspects of behavior. The area which has received most attention is probably that of the stability under uniformly distributed axial compressive forces; however, despite the large amount of attention which has been given to this problem, the present state of knowledge must be described as unsatisfactory. At the present time, predictions of critical loads usually bear little resemblance to the experimentally determined values no matter how detailed the computations are. In most analytical studies, length does not generally play a significant part, unless the cylinder is long enough to fail as a column. Similarly, calculations which are made of the number of buckles which will form around the circumference of such a shell body in compression are always in variance with the observed.

In a recent paper, Hoff[1*] suggested that the critical compressive stress and the number of buckles which occur were interrelated. His paper suggests that these parameters are a function of the test machine rigidity and the geometry and material characteristics of the specimen. It is relevant to remark that the postbuckling load deflection curve originally proposed by Kármán and Tsien is the envelope to the curves which represent the nonlinear behavior of configurations with different values of the number of buckles. From an analytical point of view, therefore, there can be little question that the variation in number of buckles with variation in geometric and loading parameters is important. Published experimental data[2] likewise shows that variation in the number of buckles with change in geometric characteristics has been noted for some considerable time; however, the consistency of the data leaves much to be desired.

A study has been made to determine the dependence of the number of circumferential buckling lobes on the geometric and mechanical properties of the test shells and the stiffness properties of the test machine. This was done

---

[*] Superscripts refer to the reference list appended to this report.

as part of a general experimental program aimed at investigating the mechanism of buckling of cylindrical shells. The prime factors which have been considered in the work reported here are the influence of L/R, R/t, and σ_{cr}/σ_{cl} on the number of circumferential buckles.

These experiments were carried out using a relatively large number of test specimens. Then the data was statistically reduced to obtain meaningful results. Over 300 shells were tested in the feasibility and research portions of this investigation.

2. OUTLINE OF THE RESEARCH PROGRAM

Experimental results obtained in questions of stability of shell bodies provide strong evidence of variability from test to test even when the specimens used are apparently identical and the test environments unaltered. There can be little doubt, therefore, that studies made of the influence of geometry and mechanical properties on buckle characteristics must be made on a statistical basis.

The first part of this test program was therefore designed to determine whether or not the distribution of buckle number would be normally distributed for a large sample of nominally identical shells. The remainder of the program was based upon studies with more restricted populations.

To determine the effect of R/t on the number of circumferential buckles, experiments were conducted for fixed values of L/R, allowing R/t to vary. Similarly, to determine the effect of L/R, tests were conducted on cylinders with fixed values of R/t, allowing L/R to vary.

Limited investigation of the influence of material modulus was made. For this purpose, shells made of brass and steel were compared.

These experiments automatically covered variation in the values of σ_{cr}/σ_{cl}.

To investigate the influence of test machine rigidity, a limited program was carried out using a rig which had been developed for a study on the influence of machine rigidity on the load carrying capability of axial compression shells. This rig is essentially a simple spring arrangement which is inserted in series with the test specimen. Identical shells were tested with varying test machine stiffness and the behaviors compared.

3. CONSTRUCTION OF THE TEST SPECIMENS

All test specimens were thin-walled circular cylindrical shells with a soldered lap joint along a generator line. They were manufactured from shimstock. The manufacturing process used was as follows:

The shimstock was cut accurately into lengths equal to the circumference of the desired cylinder, plus the overlap length. This overlap length was nominally 1/4 inch. The rectangular sheet of metal was wrapped around an accurately machined mandrel whose diameter was equal to the interior diameter of the desired shell. The edges of the sheet were overlapped, and the ends were squared. The sheet arrangement was then clamped tightly to the mandrel using elastic circumferential clamps. When this had been done, the shell was formed. Flux was applied to the joint where the two free edges met. These edges were then joined together using solder. To ensure flatness along the seam, the free edges were restrained by an extruded aluminum angle which was held securely by the circumferential clamps. The construction procedure is pictorially shown in Figure 1. Generally, the shells produced were so tight on the mandrel that it was necessary to shrink the mandrel in order to remove the specimen.

4. GEOMETRY AND MATERIAL OF SHELLS

The shells described above were manufactured from shimstock. In order that the ends should be as parallel as possible, they were manufactured in lengths which corresponded to the width of the commercial shim available. Two lengths were used. These were 6 inches and 12 inches. In the 6 inch range, specimens were made from both steel shim and brass shim, whereas, for the 12 inch cylinders only steel shim was used. Thicknesses of the shim used, varied from .002 inch to .010 inch. The exact dimensions for each of the separate shells is described in its appropriate Table of results. Figure 2 shows a shell of each radius and length tested.

The mechanical properties of the two materials were determined by normal tensile test procedures, and the characteristics obtained are given below.

$$E_{steel} = 30.5 \times 10^6 \text{ psi}$$

$$E_{brass} = 18.8 \times 10^6 \text{ psi}$$

5. TEST PROCEDURE

The test machine used in this investigation was a standard Baldwin-Lima-Hamilton 60,000 pound Universal Test Machine.

To ensure axiality of load, all shells were tested using a spherical ball loading device positioned between the top platen of the test machine and the top loading plate for the shell. See Figure 3. To eliminate edge buckling and to control the buckle position as far as possible, rings with the same diameter as the forming mandrel were inserted into the ends of the cylinders prior to testing. These rings were approximately $5/8^{th}$ of an inch in width. See Figure 4. All tests were conducted in the same test machine and care was taken to ensure that the loading platen and table were a constant distance apart. The shells were accurately centered relative to the machine. All tests were made at a constant load rate of 800 pounds per minute. The load was applied until buckling occurred. When the shell had failed, the load was held steady at the drop-off value while the buckles it had formed were measured. In shells where the complete circumference was not filled with buckles, the buckles that formed were measured. This wavelength was then used to extrapolate the buckle number to the complete circumference.

Two or more rows of buckles were observed on all the specimens; therefore, two data points on buckle number were obtained from each test shell.

6. OBSERVATIONS WITH REGARD TO BUCKLING

The failure of the steel shells was characterized by the typical snap-through action of buckling. It was noted during the test sequences that the steel shells of 6 inch lengths always buckled in a two-tier pattern located in the middle of the shell. The 12 inch steel specimens were not so consistent. In these cases, there was some tendency for buckling to occur at more random locations. Random lines of buckles would also appear on these longer specimens. Figure 5 illustrates typical buckling patterns for the 6 inch and 12 inch cylinders.

In the case of the brass shells, the two-tier snap-through behavior which had characterized the steel shells was frequently preceded by the formation of an individual buckle. This buckle progressively grew in size before catalyzing the extended buckling.

7. RESULTS

The material discussed in this section is a statistical evaluation of the data obtained from the various tests. The mean of the sample data of each series of tests was used to statistically represent that group. The sample standard deviation is also given for each of the representative populations. The mean and sample standard deviation are found in the appropriate Table of results for each family of tests.

It must be emphasized that the length, L, referred to in this section, is the effective length of the shell, i.e., the geometric length minus the width of the two end rings.

7.1 Test Series 1:

For the first series of tests, all cylinders were of the same material and geometry. Sixty-four shells were tested. The results obtained indicate that the distribution of circumferential buckle number is normal. The complete results are given in Table I and are portrayed graphically in Figures 6 and 7. It is interesting to observe that although the buckle number distribution curve is normal, the buckling load distribution is not. The distribution of load and the distribution in number of buckles for these nominally identical specimens does not appear to be correlatable.

7.2 Test Series 2:

For the second series of tests, the ratio R/t was kept constant, and the L/R ratio was varied. Since the procedures of manufacturing and testing bore a 1:1 correspondence with those used in Test Series 1, it was assumed that the distribution of circumferential buckle number would be normal in this case, also. Thus, it was decided that a smaller number of specimens than used in Test Series 1 would suffice. For this family of tests, approximately 15 specimens were used for each L/R.

The results obtained are tabulated in Table II and are graphically depicted in Figures 8 and 9. It is readily apparent from these curves that

the number of buckles decreases as the L/R ratio increases for a given R/t and E. It is apparent, too, that the ratio of the critical compressive stress to the classical critical stress also decreases as the value of L/R increases.

7.3 Test Series 3:

For the third series of tests, the L/R ratio was kept constant, and the R/t ratio was varied. It was found that the distribution of buckle number was normal as established in Test Series 1. For each value of R/t, approximately 15 cylinders were tested. The complete results are given in Table III and are displayed in Figures 10 and 11. It is clear from this data that the number of buckles produced increases as the R/t ratio increases. Over the range investigated, the increase is linear.

A limited number of specimens were tested to determine the effect on this curve of another value of L/R. These results are given in Table IV and shown in Figure 12. This evidence indicates that variation of L/R causes a bodily shift of the curve.

It is also shown that as the ratio R/t increases, the ratio of σ_{cr}/σ_{cl} decreases.

7.4 Test Series 4:

Young's modulus was varied in the fourth group of tests. This was done by comparing geometrically identical shells made of steel and brass. Twenty-eight brass cylinders with the same dimensions as the steel cylinders of Test Series 1 were tested. The results of this investigation are given in Table V. A comparison of Test Series 1 with this data shows that the modulus of the material may not play an important part in the number of buckles generated. This point may merit further investigation.

7.5 Test Series 5:

For the fifth family of tests, the shells were made of steel and were of constant R/t and L/D. The purpose of this series was to study the influence

of test machine rigidity on the buckle number. For this purpose, the basic test machine was modified. Its stiffness was varied from test to test by inclusion of an auxiliary spring system located between the base of the test specimen and the table. See Figures 13 and 14 for the auxiliary test setup.

This auxiliary device was a simple leaf spring system which had been previously used in studies of the machine rigidity effects on the initial buckling load of shells. The procedure followed in this family of tests was to test shells of geometrically and mechanically identical character in machines of varying stiffness following a random pattern. Then the results were examined with a view to determining whether or not the buckle number distribution was altered by the variation in stiffness. The results of these tests are tabulated in Table VI. They indicate that there is no change in the buckle wavelength due to a change in stiffness of the test machine. However, the extent of buckling is influenced by the stored energy in the machine.

8. CONCLUSIONS

The experiments reported in this paper show that the wavelength of the circumferential buckles for nominally identical cylindrical shells under uniformly distributed axial compression is variable. However, for a given geometry, the values obtained from a large series of tests follows a normal distribution. It is found that if the buckle number is defined as the circumference divided by the most probable wavelength, then this number is a function of both the R/t ratio and the L/R ratio for a given material. If R/t and E are kept constant, then the buckle number decreases with increasing L/R, while if L/R and E are kept constant, the buckle number increases as the ratio of R/t increases.

The limited tests performed tend to show that the modulus of the material has no influence on the buckle number. It is felt that this area should be more fully explored and a wider range of materials used than was the case in this study.

Test machine rigidity likewise appears to have no effect on the buckle number, but there is clear evidence that it is significant with regard to the extent to which the buckle pattern develops.

REFERENCES AND BIBLIOGRAPHY

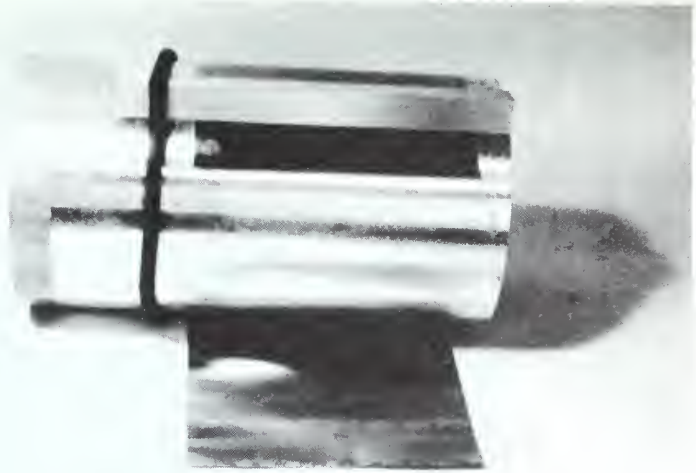
1. Hoff, N. J.: Buckling of Thin Shells, Proceedings of an Aerospace Scientific Symposium of Distinguished Lecturers in Honor of Dr. Theodore von Kármán on his 80th Anniversary, May 11, 1961, The Institute of the Aerospace Sciences, New York.

2. de Neufville, R. L.: Influence of Geometry on the Number of Buckles in Cylinders, A.I.A.A. Journal, vol. 3, no. 2, p. 364, February 1965.

3. Fung, Y. C., and Sechler, E. E.: Instability of Thin Elastic Shells, Structural Mechanics, Proceedings of the First Symposium on Naval Structural Mechanics, Edited by J. N. Goodier, and N. J. Hoff, Pergamon Press, London, 1960.

4. Bowker, L. H., and Lieberman, G. J.: Engineering Statistics, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1959.

a



b

c

d

FIG. 1. CONSTRUCTION PROCEDURE

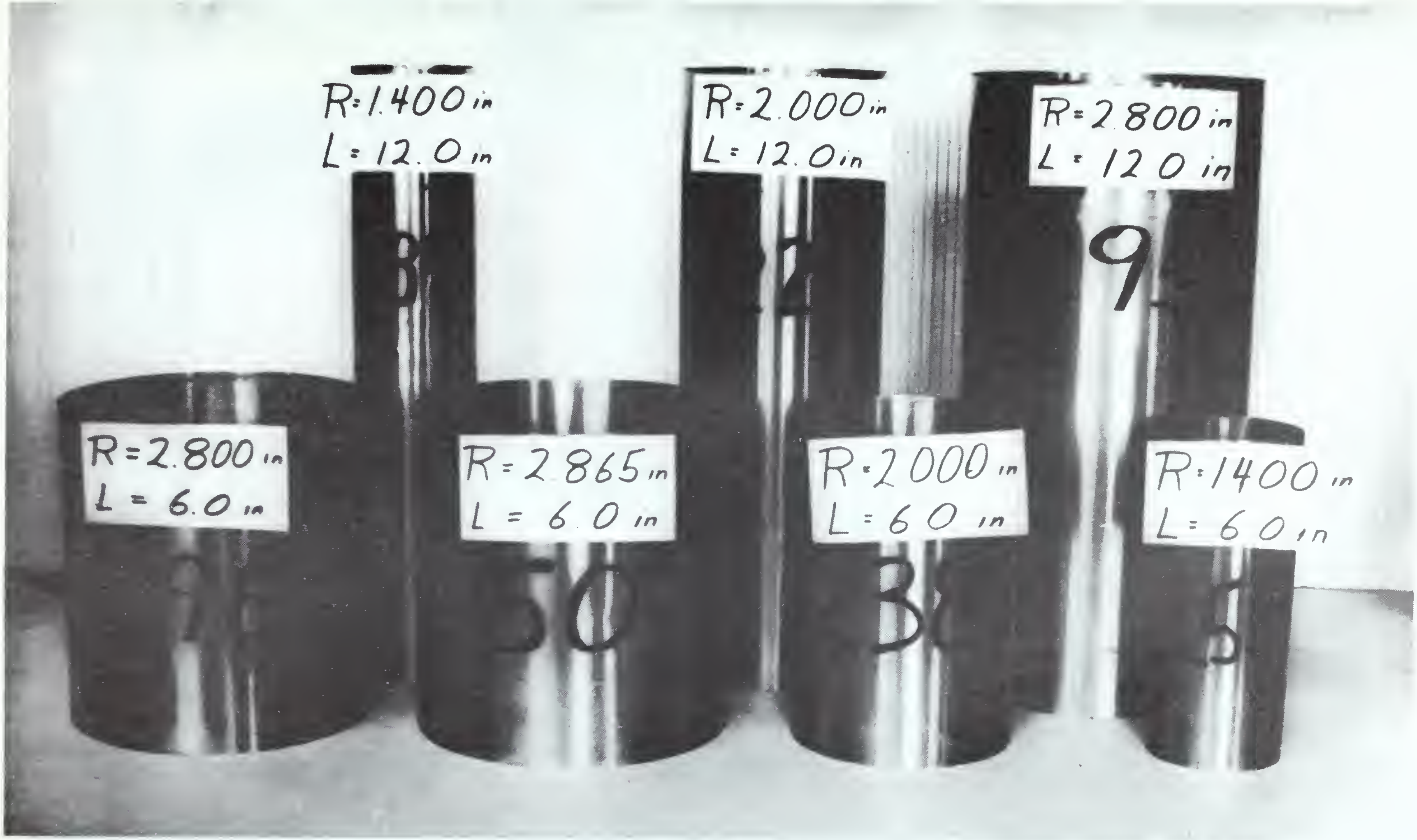


FIG. 2. VIEW OF THE SEVEN SIZES OF SHELLS

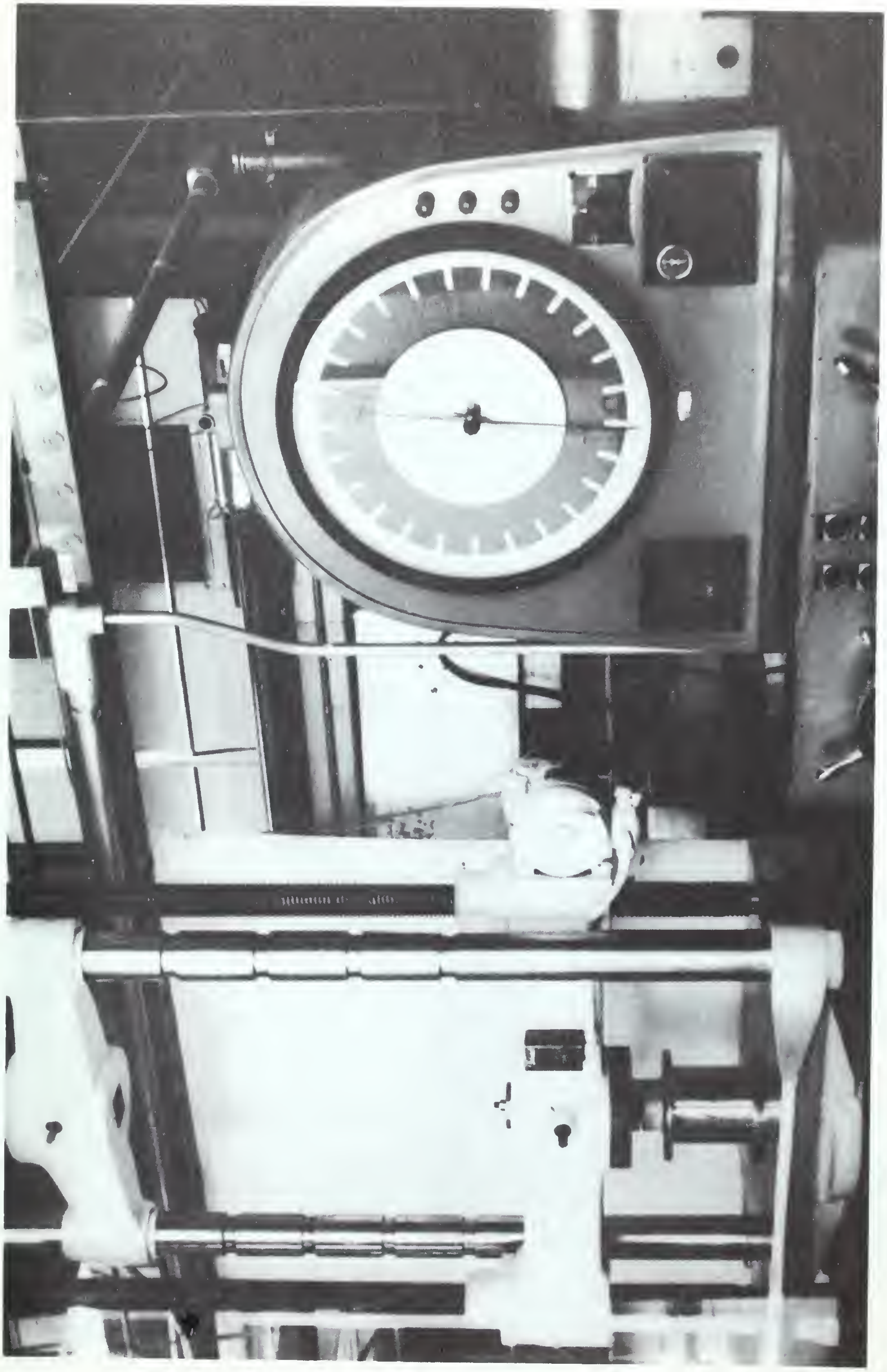
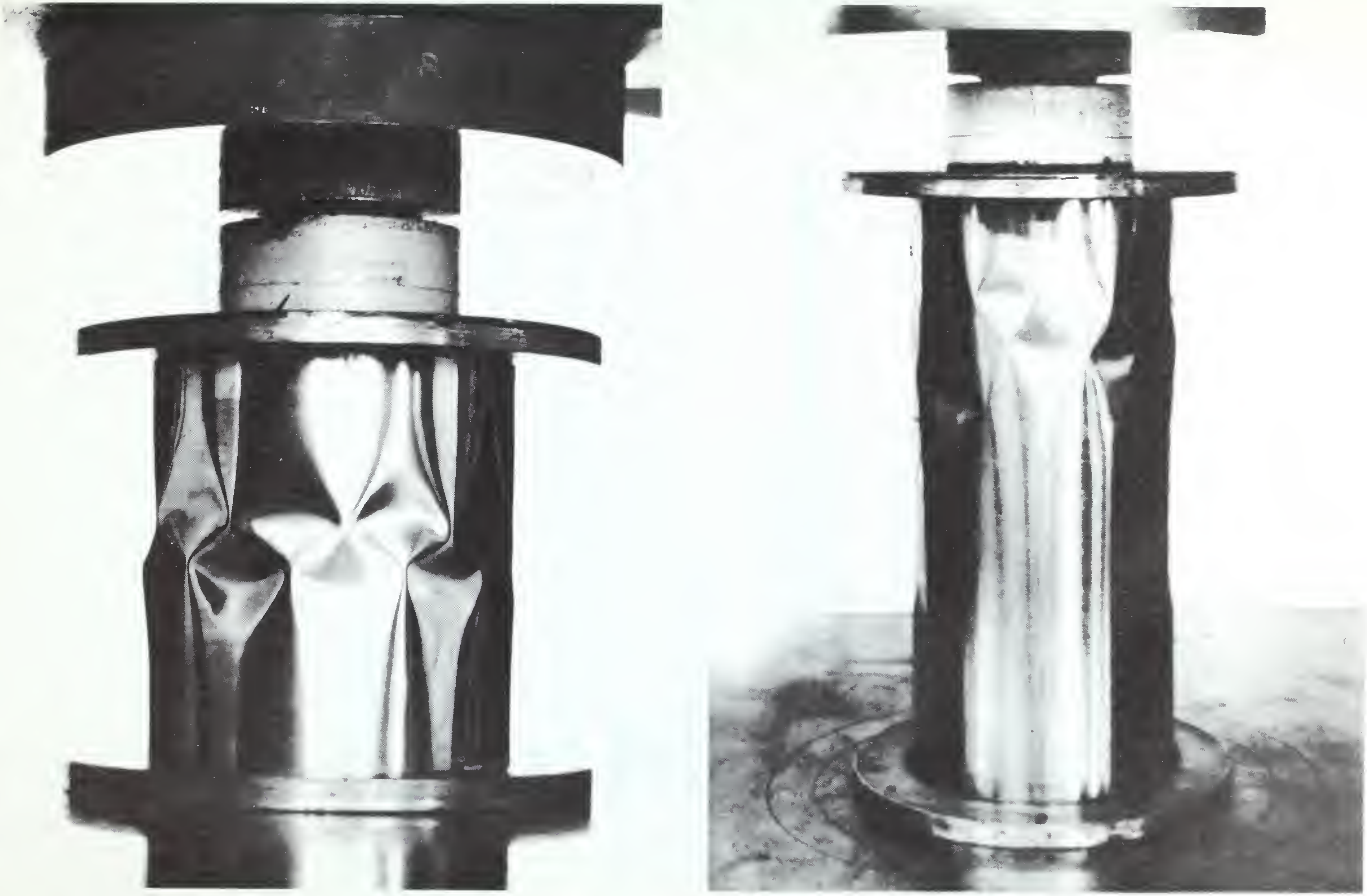


FIG. 3. TEST SETUP FOR NORMAL TESTING

FIG. 4. VIEW OF THE END RINGS



6 inch 12 inch

FIG. 5. VIEW OF TYPICAL BUCKLING PATTERNS

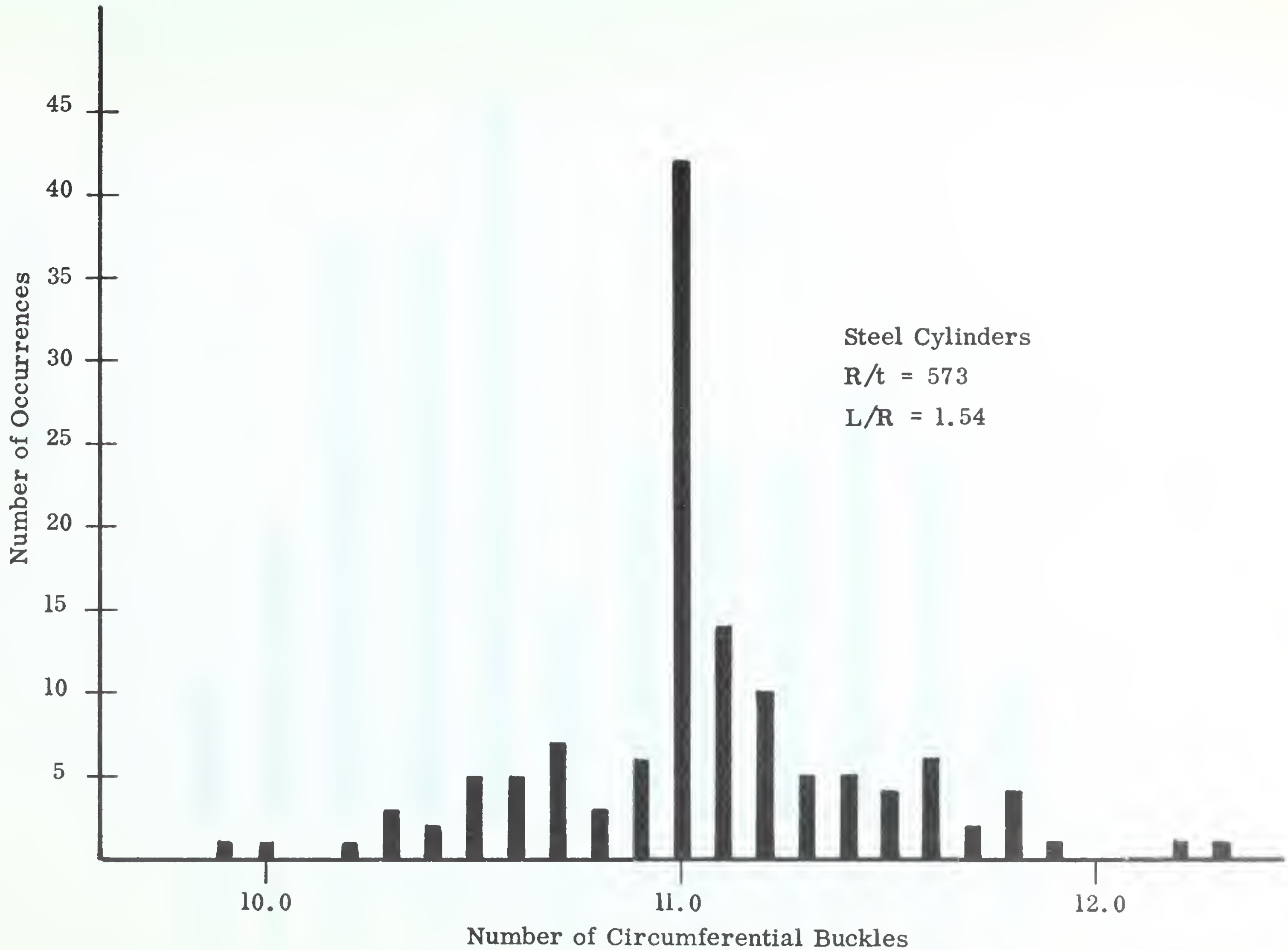


FIG. 6. FREQUENCY DISTRIBUTION OF THE NUMBER OF CIRCUMFERENTIAL BUCKLES

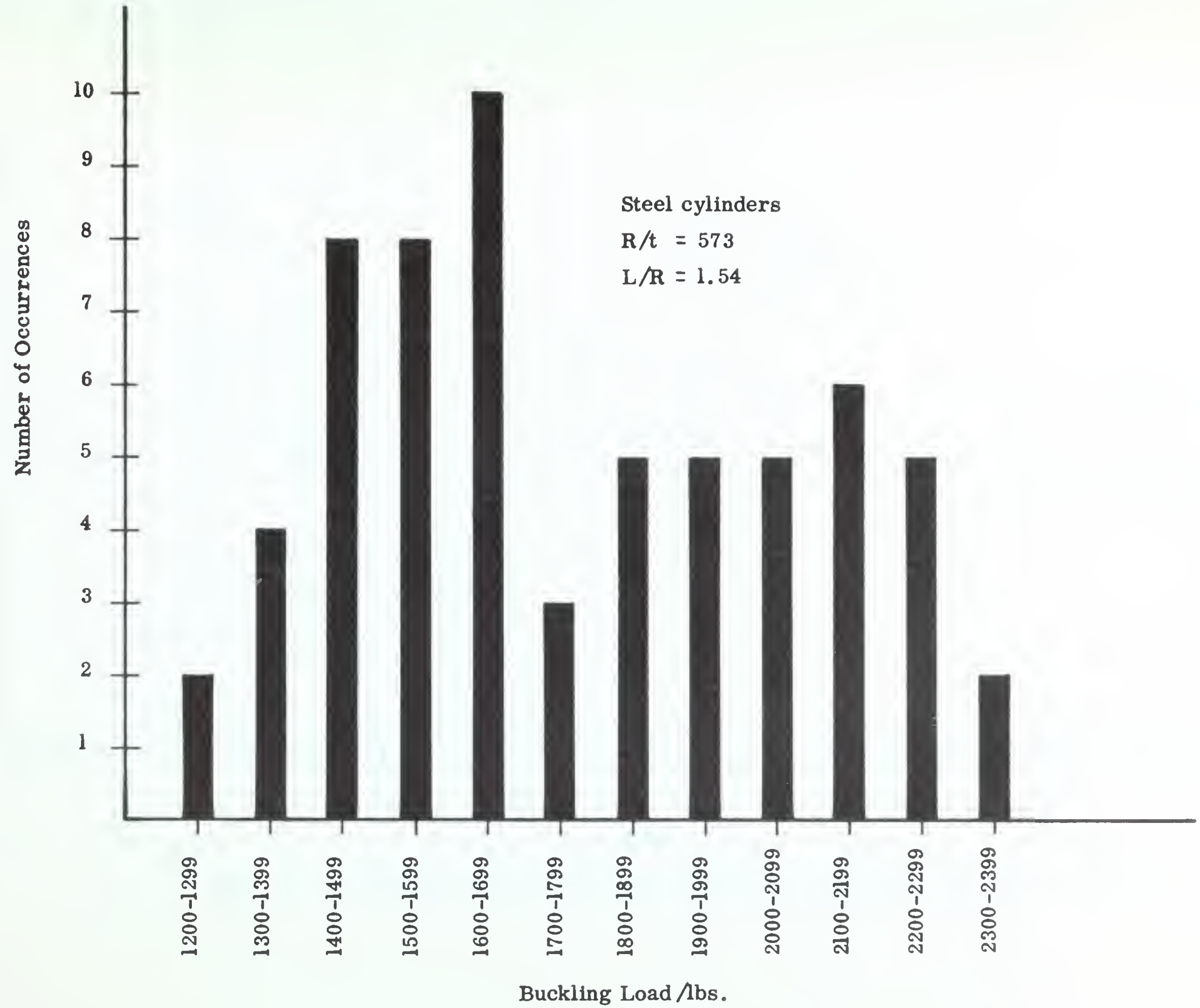


FIG. 7. FREQUENCY DISTRIBUTION OF THE BUCKLING LOAD

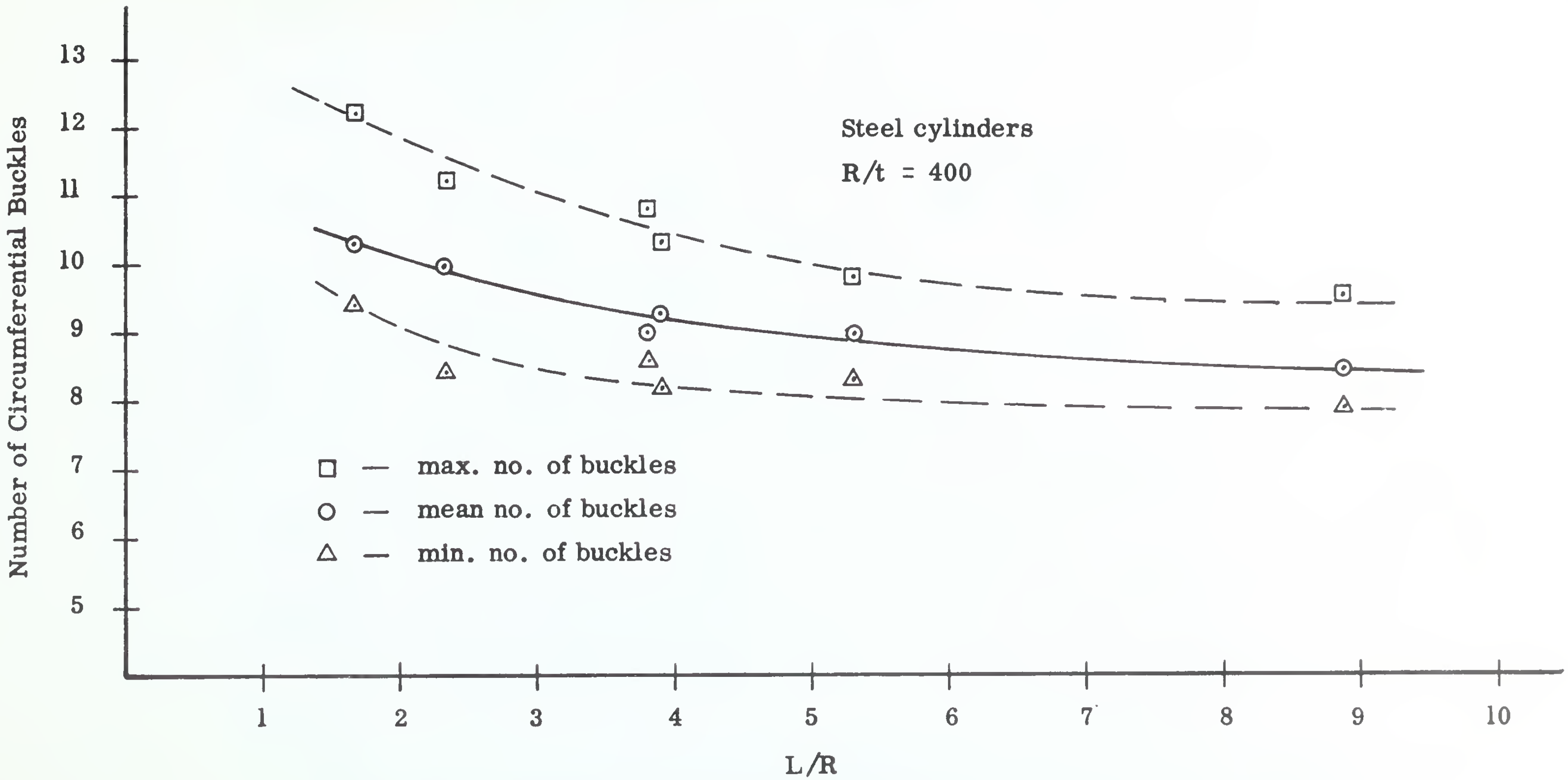


FIG. 8. NUMBER OF CIRCUMFERENTIAL BUCKLES VS. L/R

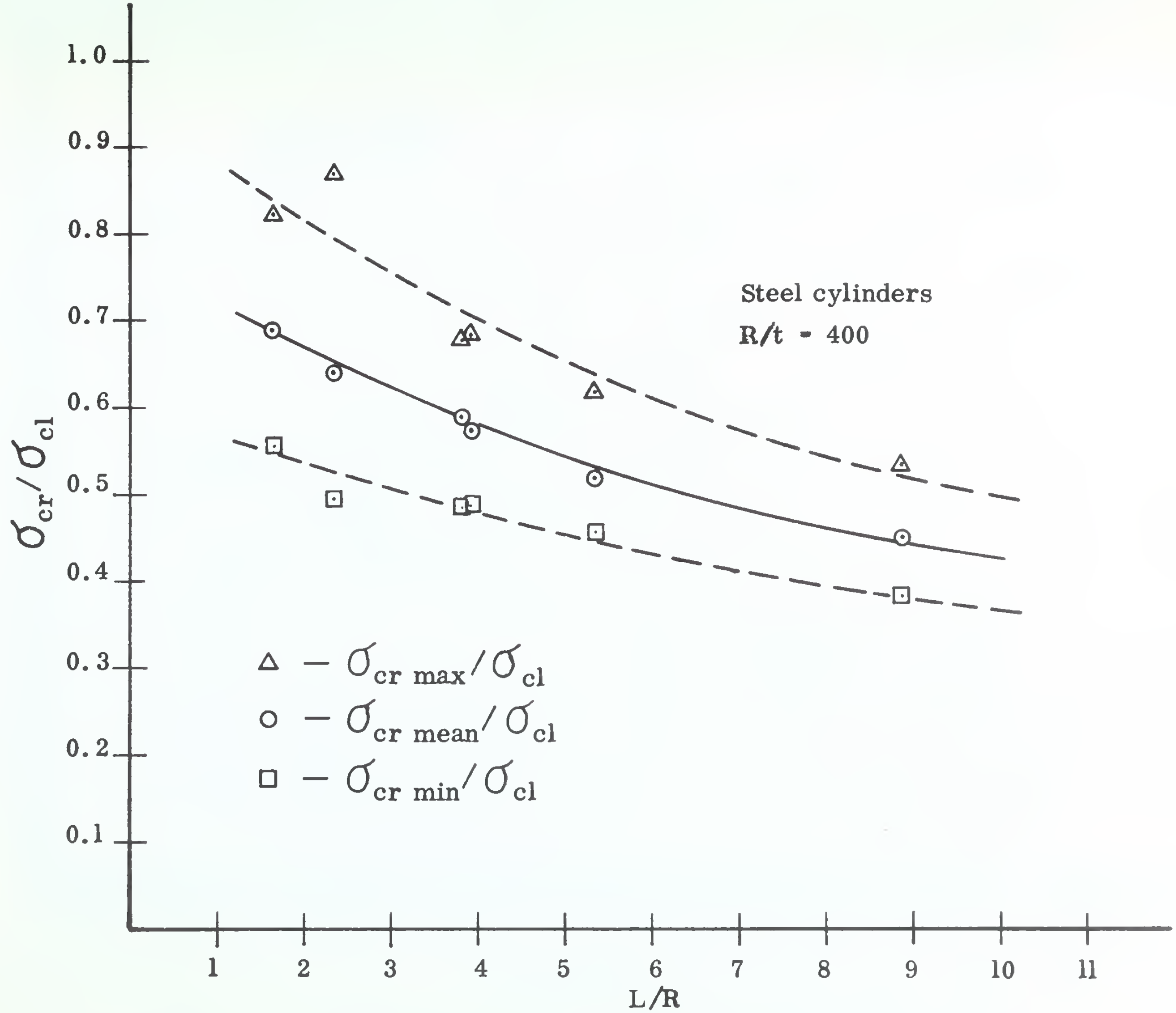


FIG. 9. σ_{cr}/σ_{cl} VS. L/R

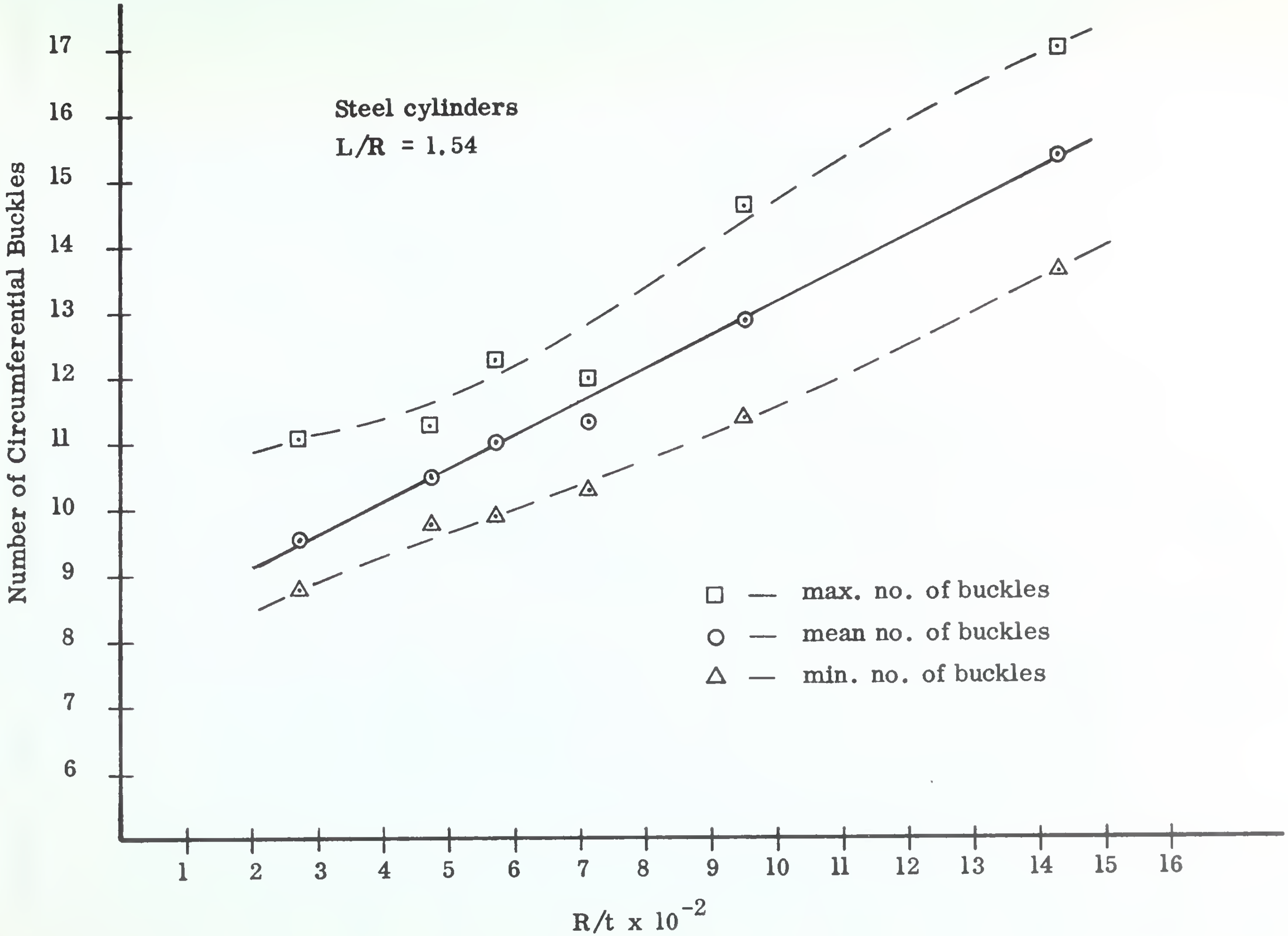


FIG. 10. NUMBER OF CIRCUMFERENTIAL BUCKLES VS. R/t

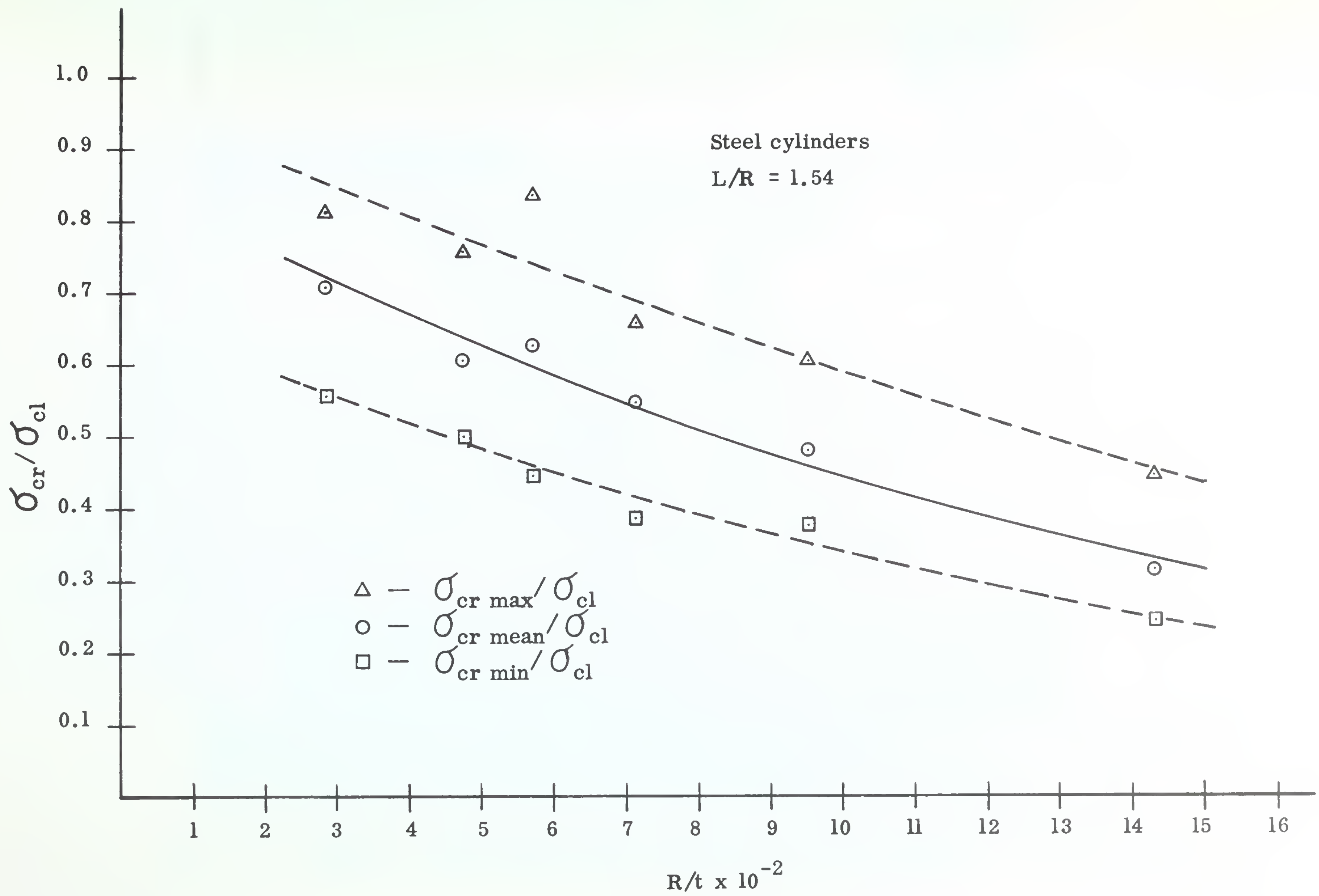


FIG. 11. σ_{cr}/σ_{cl} VS. R/t

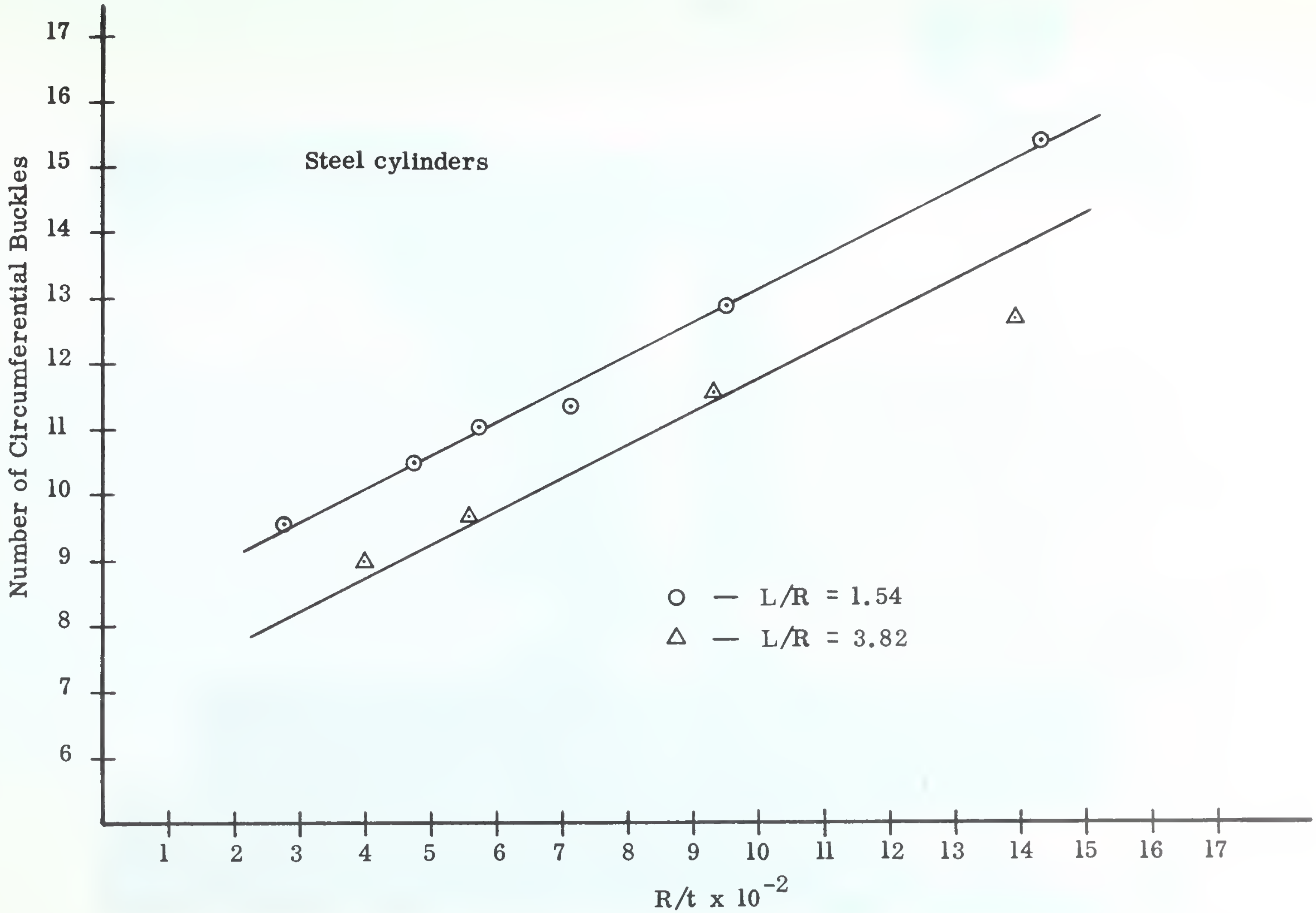


FIG. 12. NUMBER OF CIRCUMFERENTIAL BUCKLES VS. R/t FOR TWO VALUES OF L/R

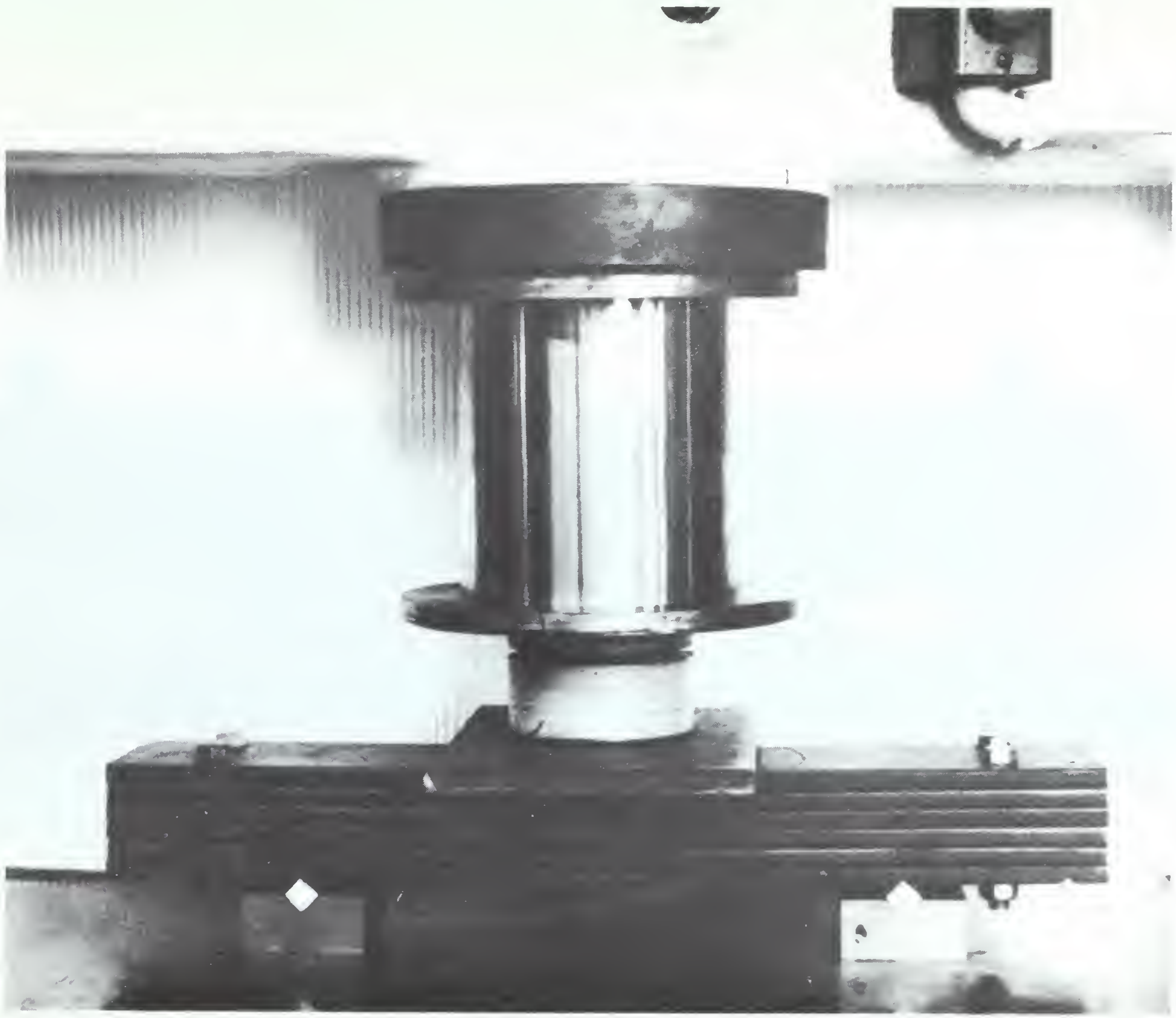


FIG. 13. TEST SETUP WITH SPRING SYSTEM INSERTED

FIG. 14. CLOSE-UP VIEW OF THE SPRING

TABLE 1

TEST SERIES 1: TO CHECK THE NORMALITY OF CIRCUMFERENTIAL BUCKLE NUMBER

Steel Cylinders $R/t = 573$ $L/R = 1.54$

$t = .005$ in. $R = 2.865$ in.

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.
1	11.2 11.2	1415
2	11.3 10.9	1490
3	11.2 11.6	1660
4	11.6 11.5	1635
5	10.6 10.6	1615
6	10.3 10.0	1845
7	11.0 11.0	1805
8	10.5 10.8	2100
9	11.1 11.0	2360
10	11.0 11.0	1820
11	11.0 11.0	2180
12	11.0 10.9	1450
13	11.0 11.0	1540
14	11.0 11.0	1650
15	10.7 10.7	1755
16	10.9 11.5	---
17	11.8 11.6	1505

TABLE 1 (Cont'd)

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.
18	11.0 11.0	1640
19	11.0 11.0	2195
20	11.1 11.0	2065
21	11.0 11.0	1645
22	10.8 11.2	1325
23	11.0 11.0	1565
24	10.4 10.9	1255
25	11.6 11.8	1425
26	11.1 11.4	1510
27	11.5 11.6	1770
28	11.0 12.3	2150
29	11.7 11.8	1440
30	10.3 11.0	1690
31	11.4 11.2	1940
32	11.4 11.3	1910
33	11.2 11.0	1490
34	11.0 11.0	1465
35	10.5 10.7	1420
36	11.0 11.0	1540

TABLE 1 (Cont'd)

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.
37	11.3 11.4	2080
38	11.3 11.8	1510
39	11.7 11.1	1360
40	11.5 11.1	1250
41	11.2 11.3	1365
42	11.6 11.1	1380
43	11.1 10.5	1610
44	11.0 11.0	1700
45	10.8 10.9	1935
46	10.7 10.9	1695
47	10.6 10.9 11.1	2055
48	10.7 10.2	1630
49	11.0 11.0	1950
50	11.0 11.1	2150
51	11.0 11.0	2210
52	11.2 11.1	1595
53	11.9 12.2	1570
54	11.4 11.1	1845

TABLE 1 (Cont'd)

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.
55	10.7 11.1	1990
56	9.9 10.3	2275
57	11.2 11.0	2095
58	10.5 11.2	2200
59	11.0 11.1	2330
60	10.4 10.6	2260
61	10.5 10.7	2190
62	11.0 11.1	2225
63	11.0 11.0	2000
64	11.0 11.0	1870
Average	11.05	1771

Sample Standard Deviation = .364 buckles

TABLE II

TEST SERIES 2: TO DETERMINE THE EFFECT OF L/R ON CIRCUMFERENTIAL BUCKLE NUMBER

Steel Cylinders R/t = 400 L/R Variable

GROUP 1

t = .007 in. R = 2.800 in. L/R = 1.67

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	10.6 10.7 11.0	3650	1220
2	9.7 9.8	4060	1200
3	10.4 10.7	3910	1240
4	9.4 12.2	4550	1200
5	11.0 10.4	4520	1360
6	9.7 10.0 9.8	3940	1230
7	9.7 9.7	3710	1220
8	10.0 10.0	3160	1260
9	10.0 9.5	3200	1245
10	10.8 9.8	4520	1350
11	10.5 11.3	3975	1260
12	10.0 9.4	3135	1120
13	10.4 10.3	3055	1360
14	10.5 10.5 10.8	4035	1260

GROUP 1 (Cont'd)

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
15	10.4 10.4	3745	1250
Average	10.28	3811	1252

Sample Standard Deviation = .680 buckles

GROUP 2

t = .005 in. R = 2.000 in. L/R = 2.34

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	8.4 9.8	1780	620
2	10.3 10.3	1580	570
3	9.4 11.2	2210	575
4	9.6 10.3	1725	525
5	9.6 10.0	1940	700
6	10.1 10.4	1610	605
7	9.3 10.5	1410	630
8	9.6 9.6	1395	550
9	11.2 10.5	2450	745
10	10.4 9.8	1395	520
11	10.3 9.7	1660	770
12	9.0 10.1	1440	520
13	10.4 10.1 10.1	2130	630
14	10.6 9.1	---	---
15	9.4 9.5	2460	640

GROUP 2 (Cont'd)

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
16	9.8	1920	650
	10.1		
Average	9.95	1807	617

Sample Standard Deviation = .669 buckles

GROUP 3

t = .007 in. R = 2.800 in. L/R = 3.82

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	9.2	3210	1120
	9.4		
	10.8		
2	8.6	2850	1050
	9.4		
	8.8		
3	9.6	3730	1070
	9.4		
4	8.3	2790	1130
	8.8		
5	8.4	3480	1130
	8.2		
6	8.4	2670	1220
	8.6		
Average	8.99	3122	1120

Sample Standard Deviation = .700 buckles

GROUP 4

t = .003 in. R = 1.200 in. L/R = 3.91

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	9.4	590	205
	9.9		
2	9.0	550	210
	9.6		
3	8.3	600	195
	8.8		
4	7.8	495	195
	9.4		
5	10.0	515	215
	10.0		
6	8.2	540	195
	8.6		

GROUP 4 (Cont'd)

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
	t = .003 in.	R = 1.200 in.	L/R = 3.91
7	10.0 9.7	530	225
8	9.0 9.2 9.6	595	205
9	9.2 8.2	690	205
10	10.0 9.3	550	195
11	7.9	605	220
12	9.1 8.7	625	190
13	9.9 9.1	570	220
14	9.1 9.5	650	200
15	10.1 10.3	690	220
Average	9.23	590	206

Sample Standard Deviation = .685 buckles

GROUP 5

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
	t = .005 in.	R = 2.000 in.	L/R = 5.34
1	8.4 8.4	1510	540
2	8.8 9.7	1440	510
3	9.6 9.4	1430	570
4	9.8 9.2	1580	580
5	8.6 8.8	1525	530
6	8.6 9.1	1745	570

GROUP 5 (Cont'd)

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
t = .005 in.	R = 2.000 in.	L/R = 5.34	
7	9.1 9.4	1640	595
8	9.1 9.2	1530	575
9	8.4 8.4	1460	495
10	8.4 9.1	1390	530
11	9.6 8.7	1335	530
12	9.1 9.1	1290	465
13	9.1 9.1	1290	590
14	9.3 9.5	1460	475
15	8.7 8.7	1370	540
16	8.6 8.3	1510	460
Average	8.98	1469	535

Sample Standard Deviation = .497 buckles

GROUP 6

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
t = .003 in.	R = 1.2000 in.	L/R = 8.9	
1	8.3 8.2	485	195
2	8.4 8.6	435	235
3	8.4 8.1	400	210
4	9.1 8.6	540	160
5	9.1 8.6	445	180

GROUP 6 (Cont'd)

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
	t = .003 in. R = 1.2000 in. L/R = 8.9		
6	8.4 8.2	530	170
7	7.9 8.0	485	170
8	8.5 8.2	445	190
9	8.2 9.5	495	155
10	7.9 8.1	495	145
11	8.1 8.1	410	195
12	8.2 8.6	485	170
13	8.3 8.1	435	175
14	8.6 9.3	430	155
15	8.2 9.3	410	140
16	8.6 8.6	390	190
Average	8.45	457	177

Sample Standard Deviation = .350 buckles

TABLE III

TEST SERIES 3: TO DETERMINE THE EFFECT OF R/t ON CIRCUMFERENTIAL BUCKLE NUMBER

Steel Cylinders R/t Variable L/R = 1.54

GROUP 1

t = .010 in. R = 2.865 in. R/t = 286

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	9.2 9.3	6260	2880
2	9.8 9.9	8860	2800
3	9.2 9.6	8640	2480
4	9.4 9.3	8780	2400
5	9.8 10.3	9080	2440
6	9.1 9.4	6580	2400
7	10.1 11.1	7080	2800
8	10.1 9.3 9.3	9180	2520
9	9.8	8100	2380
10	8.8 9.2	6640	2510
11	9.0 8.8	8080	2600
12	9.2 9.5	8600	2580
13	9.4 9.6	8990	2420
14	9.4 10.0	7390	2590
Average	9.53	8019	2557

Sample Standard Deviation = .534 buckles

GROUP 2

t = .006 in. R = 2.865 in. R/t = 478

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	10.3 10.7	3075	920
2	10.1 10.3	2230	1030
3	10.0 10.9	2160	1100
4	10.2 10.8	2180	990
5	10.7 10.8	2030	1140
6	9.9 9.8	2290	890
7	11.3 11.2	2540	790
8	10.7 10.1	2760	820
9	10.3	2360	840
10	10.4 9.8	2640	830
11	10.3 10.3	2410	800
12	10.7 11.1	2510	815
13	10.7 11.0	2080	800
14	10.7 10.8	2980	900
15	10.1 10.5	2730	780
Average	10.47	2465	896

Sample Standard Deviation = .356 buckles

GROUP 3

t = .005 in. R = 2.865 in. R/t = 573

See TABLE I

Average Number of Buckles = 11.05

Sample Standard Deviation = .364

GROUP 4

t = .004 in. R = 2.865 in. R/t = 717

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	11.2 10.3	925	350
2	11.6 11.3	860	395
3	11.4 11.4	1110	350
4	11.4 12.0	700	500
5	12.0 11.8	885	360
6	11.8 11.1 11.3	1140	390
7	11.7 11.2 11.3	1185	350
8	11.3 11.5	940	415
9	11.0 11.0	890	400
10	11.1 10.5	1160	360
11	11.4 11.4	1070	350
12	11.6 11.3	925	370
13	11.1 10.8	1100	400
Average	11.31	992	384

Sample Standard Deviation = .498 buckles

GROUP 5

t = .003 in. R = 2.865 in. R/t = 957

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	12.6 12.3	435	200
2	12.8 12.9	450	230
3	13.0 12.9	460	210
4	12.3 12.0	470	230
5	11.4 12.4	540	180
6	12.3 12.4	570	250
7	12.9 12.6	450	305
8	12.2 12.0	580	220
9	11.4 12.3	610	225
10	13.2 12.6	500	160
11	13.6 12.5	---	---
12	12.5 13.1	435	250
13	13.4 14.6	380	225
14	14.1 14.0	535	240
15	14.9 15.5	420	270
Average	12.89	488	228

Sample Standard Deviation = .954 buckles

GROUP 6

t = .002 in. R = 2.865 in. R/t = 1433

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	14.7 13.5	150	75
2	16.3 16.0	130	80
3	13.6 14.1	140	70
4	14.4 14.3	120	70
5	16.5 15.4	140	90
6	15.6 16.3	110	80
7	16.4 17.0	140	60
8	15.3 15.7	165	--
9	14.9 15.4	185	--
10	14.3 14.1	200	--
11	15.2 15.2	135	--
12	14.8 15.6	165	--
13	15.0 16.5	130	--
14	16.0 14.8	130	--
15	16.7 17.0	130	--
16	15.0 14.8	135	--
Average	15.33	144	75

Sample Standard Deviation = .873 buckles

TABLE IV

TEST SERIES 3a: FURTHER TESTS ON THE EFFECT OF R/t WITH A DIFFERENT VALUE OF L/R

Steel Cylinders   R/t Variable   L/R = 3.82

GROUP 1

t = .002 in.   R = 2.800 in.   R/t = 1398

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	11.3 11.3	125	50
2	12.8 13.2	115	75
3	13.6 13.7	110	60
4	12.4 12.8	130	80
5	12.5 12.8	105	70
Average	12.64	117	67

Sample Standard Deviation = .824 buckles

GROUP 2

t = .003 in.   R = 2.800 in.   R/t = 934

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	11.1 11.1	310	140
2	12.3 11.7	375	170
3	11.7 12.3	270	---
4	11.0 10.8	360	175
5	11.9 11.4	420	160
Average	11.53	347	161

Sample Standard Deviation = .536 buckles

GROUP 3

t = .005 in. R = 2.800 in. R/t = 560

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	10.7	1685	470
	10.6		
	9.8		
2	9.9	1115	630
	9.8		
3	9.3	925	570
	8.8		
4	9.7	1250	505
	10.2		
5	9.5	1800	465
	9.4		
6	9.0	1430	425
	9.0		
Average	9.67	1304	511

Sample Standard Deviation = .591 buckles

GROUP 4

t = .007 in. R = 2.800 in. R/t = 400

See TABLE II

Average Number of Circumferential Buckles = 8.99

TABLE V

TEST SERIES 4: TO DETERMINE THE EFFECT OF YOUNG'S MODULUS ON CIRCUMFERENTIAL BUCKLE NUMBER

$R/t = 573$ $L/R = 1.54$

$R = 2.865$ in. $t = .005$ in.

GROUP 1

Brass Cylinders

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	11.1 10.9	525	---
2	9.6 9.4	670	---
3	12.0 10.8	740	---
4	12.5	730	---
5	12.0 12.0	745	---
6	12.0 11.7	865	---
7	11.9 10.0	615	---
8	11.9 10.4	530	---
9	12.0 11.7	760	---
10	10.8 11.1	795	375
11	11.4 11.7	850	370
12	11.4 11.8	945	290
13	11.6 11.6	850	310
14	10.9 11.4	1010	390
15	10.6 10.8	740	315
16	10.9 10.9	710	345

GROUP 1 (Cont'd)

Test Number	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
17	10.9 12.0	905	315
18	10.8 11.5	805	370
19	11.1 11.4	650	380
20	11.4 11.1	930	315
21	11.8 11.2	630	340
22	11.3 11.1	770	380
23	11.4 11.8	935	360
24	10.6 11.1	760	360
25	10.4 11.1	920	355
26	11.4 11.7	720	410
27	11.8 12.0	820	360
28	11.1 10.4	1140	340
Average	11.3	788	352

Sample Standard Deviation = .511 buckles

GROUP 2

Steel Cylinders

See TABLE I

Average Number of Circumferential Buckles = 11.05

TABLE VI

TEST SERIES 5: TO DETERMINE THE EFFECT OF TEST MACHINE STIFFNESS ON CIRCUMFERENTIAL BUCKLE NUMBER

Steel Cylinders $R/t = 573$ $L/R = 1.54$

$t = .005$ in. $R = 2.865$ in.

Test Number	Spring Device Condition	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
1	6 springs bolts loose	10.4 12.3 10.8	1280	420
2	" "	10.3 10.6	1610	435
3	" "	10.3 11.1	1640	420
4	" "	10.6 10.9	1920	475
5	" "	10.8 9.8	1615	320
6	" "	11.6 11.3 11.0	1530	410
7	" "	10.5 10.9	1590	550
	Average	10.8		
8	6 springs bolts tight	11.0 11.0	1770	600
9	" "	10.7 11.4	1835	530
	Average	11.0		
10	5 springs bolts loose	10.1 10.7	2100	490
11	" "	11.0 10.1	---	---
	Average	10.5		

TABLE VI (Cont'd)

Test Number	Spring Device Condition	Number of Circumferential Buckles	Buckling Load/lbs.	Postbuckling Load/lbs.
12	3 springs	10.6	1330	470
	bolts loose	10.8		
13	" "	11.0	1250	460
		10.4		
	Average	10.7		
14	1 spring	10.5	1320	330
		10.7		
15	" "	10.3	1380	370
		10.7		
	Average	10.6		